State Diagrams

Discrete Structures (CS 173)

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Slides based on Derek Hoiem and Elsa Gunter, University of Illinois
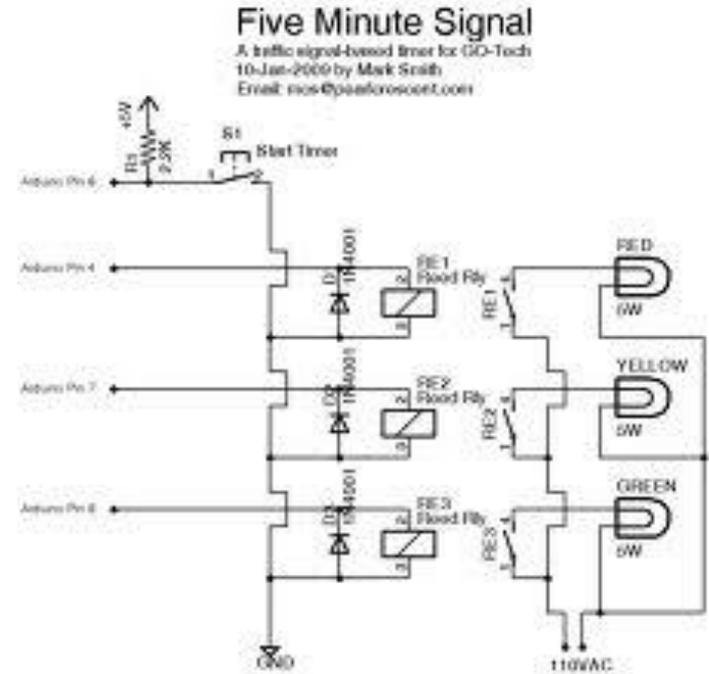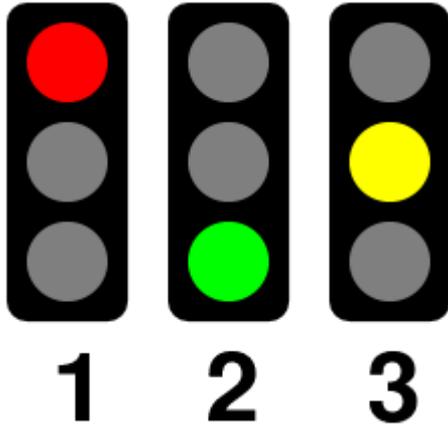
State diagrams

- Model set of valid actions and their effect on "world" state
  - Nodes represent states
  - Edges indicate that it is possible to transition from one state to another
  - Labels on edges indicate how to transition

- Many applications
  - Simple electronics
  - Games, e.g., AI controls
  - Probabilities over multiple turns
  - Probabilistic inference for multiple variables (e.g., handwriting recognition)
  - Planning in robotics

State diagrams

Transition function:
$(\text{dining rm}, \text{south}) \rightarrow \text{hall}$
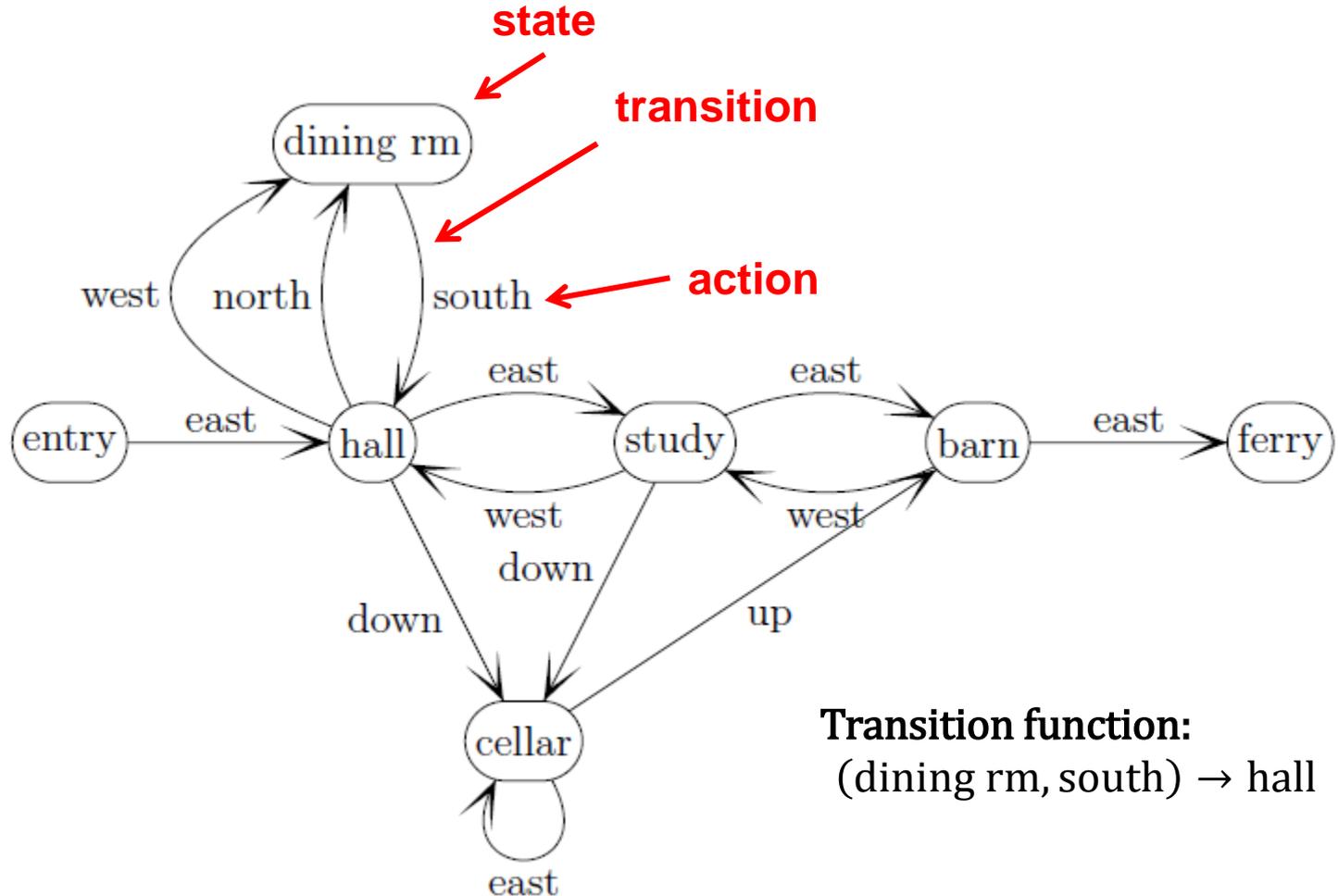
Simple example: traffic signal

Transition functions and state diagrams

States: Village, Rock, Snake, Chasm, Gold, Desert

Transitions:

(Village, North) → Desert

(Village, East) → Snake

(Desert, South) → Village

(Desert, East) → Rock

(Rock, West) → Desert

(Snake, West)→Village

(Snake, East) → Chasm

(Snake, South) → Gold

(Gold, North)→Snake

State diagrams and counting

Suppose you have 3 red pills and 2 blue pills in a pouch. You draw three of them at random. What is the probability that you have exactly one blue pill?

Dice games

What is the chance of rolling snake eyes (double ones) in one roll of two six-sided dice?

If I am allowed to re-roll, what is the probability of getting 1-1?

If I have n dice and a person is trying to roll as many ones as possible in m rolls, what is the computational complexity of calculating the odds of each outcome?

Challenge problem

If Joe is going for "Yahtzee" (five of a kind with five dice), what is the chance that he will get it within two re-rolls?
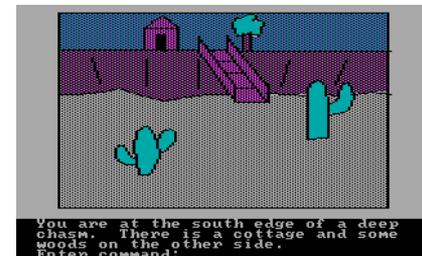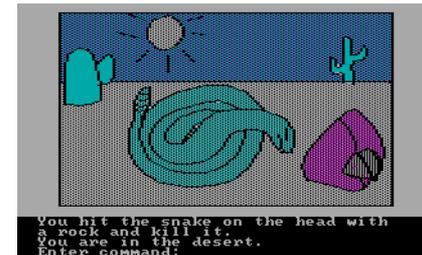
Suppose we roll a two-sided die until the sum is a non-zero multiple of 3. What is the chance of getting there in three or fewer rolls?

Things to remember

- Permutation and Combination problems can be broken down into sub-problems of selection and permutation

- When elements are drawn uniformly at random, $P(x)$ is number of ways to make x divided by total number of combinations
  - E.g., 1/6 chance of rolling "7" total with two dice because there are 6 ways to roll "7" and 36 possible rolls

- State diagrams are helpful for calculating odds when multiple turns are involved
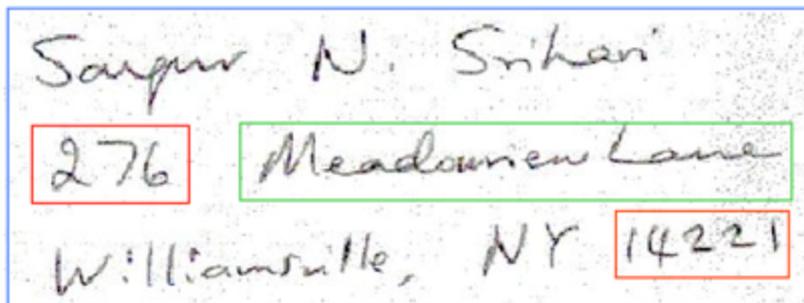
Transition functions and state diagrams

Input sequence to beat the game: N, E, E, E, E, N, E, N, W, GET ROCK, N, W, N, W, N, W, N, THROW ROCK, N, DRINK WATER, E, GET STICK, THROW STICK, W, N, THROW STICK, LOOK HOLE, GET NOTE, N, W, LIFT ROCK, N, GET NOTE, E, GET LOCKET, E, E, S, W, W, LOOK HOLE, GET CRACKER, E, N, N, W, N, W, N, SAY HOCUS, N, GO HOUSE, GET APPLE, W, N, LOOK GNOME, N, E, SAY HISS, GO CREVICE, S, S, S, GET BREAD, GET LOCKET, GET CRACKER, UNLOCK DOOR, OPEN DOOR, GO DOOR, U, GO HOLE, N, E, S, GIVE CRACKER, GET VIAL, N, W, S, W, W, THROW BREAD, N, GET ROPE, GO BOAT USE BLANKET, N, N, DRINK WATER, N, E, E, E, GO BEACH, N, N, E, GET ANCHOR, W, TIE ROPE, TO ANCHOR, THROW ANCHOR, UP, GET SHOVEL, DOWN, S, S, DIG X, LOOK TREASURE, GRAB CHEST, LEAVE, E, N, W, GO CAVE, OPEN CHEST, LOOK CHEST, GET HARP, N, E, N, DRINK VIAL, FLY NORTH, N, GET RING, N, W, FOLLOW RAINBOW, GET COIN, N, SAY LUCY, W, W, N, GO CAVE, GET ALL, N, S, W, PLAY HARP, N, N, BUY HORN, N, N, BLOW HORN, N, U, E, OPEN CLOSET, LOOK CLOSET, GET SHOES, LOOK SHOES, W, D, W, W, LOOK THRONE, THROW APPLE, N, E, LOOK CABINET, PICK LOCK, WITH KNIFE, OPEN DOOR, E, U, D, U, WEAR RING, RUB RING, D, E, KISS FROG, WEAR SHOES, SAY WHOOSH
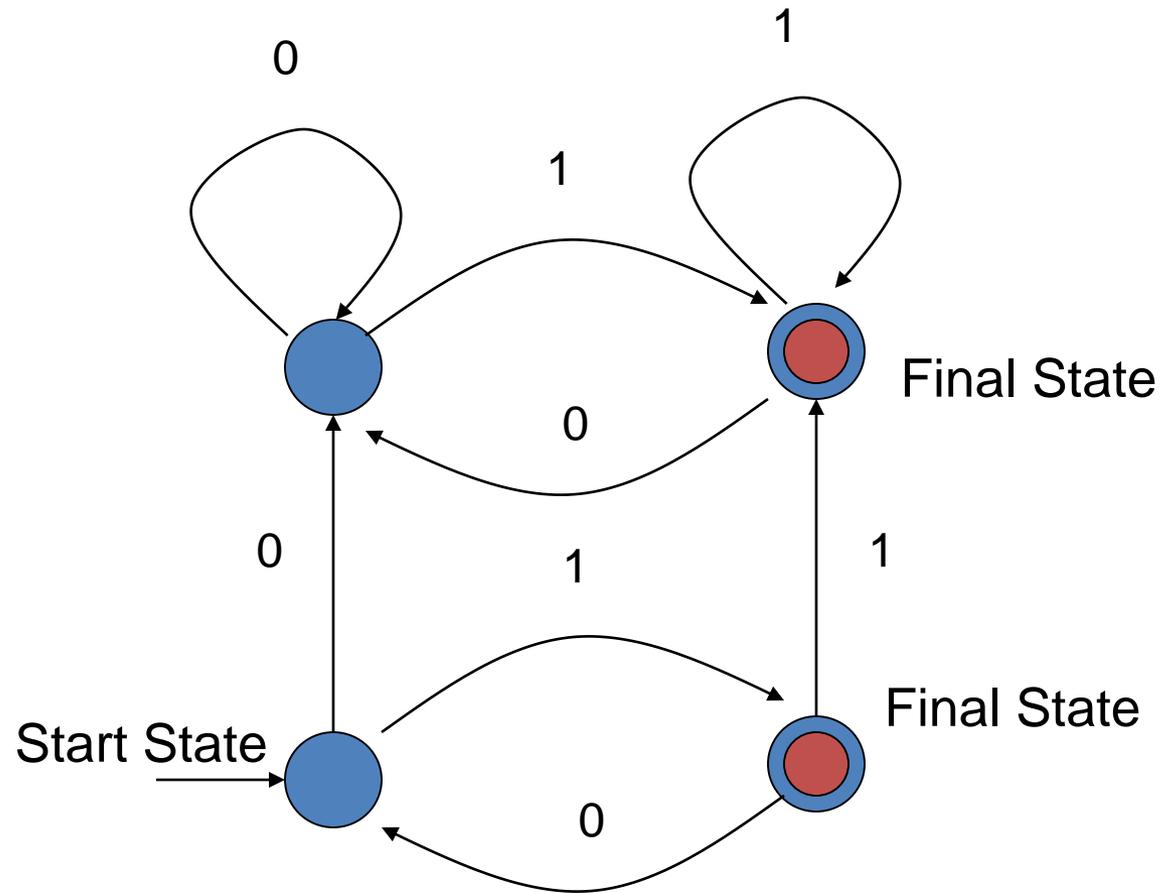

Adventures in Serenia

Handwriting recognition

Finite State Automata

- A finite state automata over an alphabet is:
  - a directed graph
  - edges are labeled with elements of alphabet, or empty string
  - some nodes (or *states*), marked as final
  - one node marked as start state

- Syntax of FSA

Example FSA

Deterministic FSA's

- If FSA has for every state *at most one edge* for each letter in alphabet then FSA is *deterministic*

  - No edge labeled with ε

- In general FSA in *non-deterministic:*

  - There may be more than one edge for some letter in alphabet.

  - NFSA also allows edges labeled by ε

- Deterministic FSA special kind of non-deterministic FSA

DFSA Language Recognition

- Think of a DFSA as a board game; DFSA is board

- You have string as a deck of cards; one letter on each card

- Start by placing a disc on the start state

DFSA Language Recognition

- Move the disc from one state to next along the edge labeled the same as top card in deck; discard top card

- When you run out of cards,
  - if you are in final state, you win; string is in language
 - if you are not in a final state, you lose; string is not in language

DFSA Language Recognition -Summary

- Given a string over alphabet
- Start at start state
- Move over edge labeled with first letter to new state
- Remove first letter from string
- Repeat until string gone
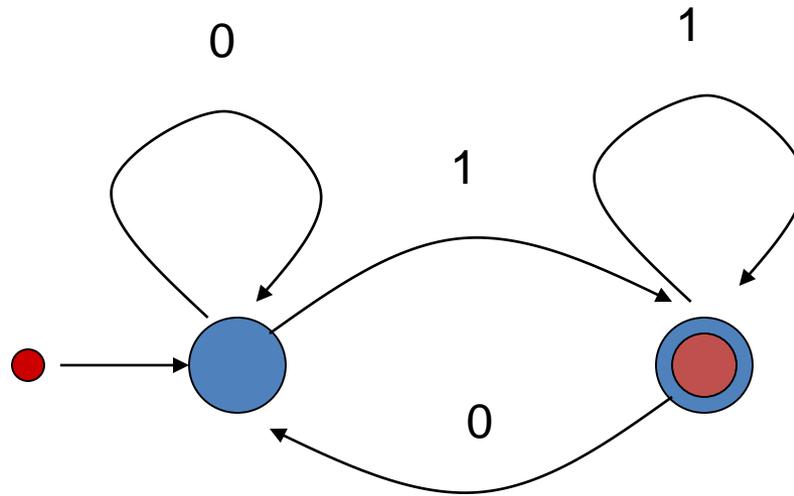- If end in final state then string in language

- Semantics of FSA

Example DFSA

- Regular expression: $(0 \vee 1)^* \, 1$
  - This says 0 or 1 may be repeatedly 0 or (arbitrarily many times), followed by a 1.

- Deterministic FSA
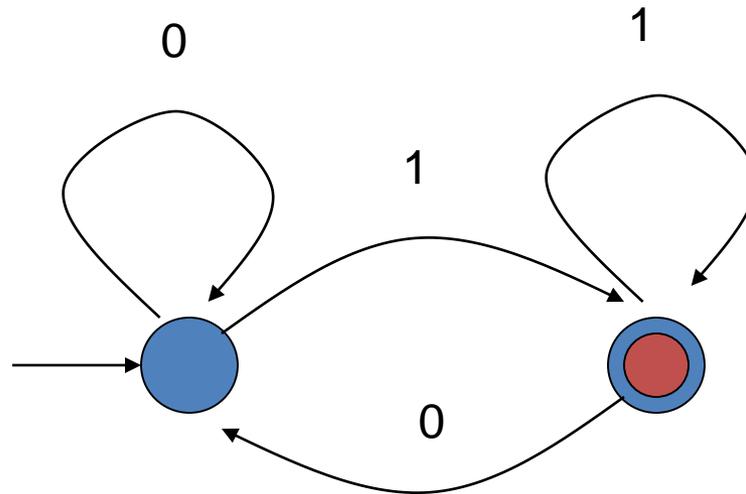
Example DFSA

- Regular expression: $(0 \vee 1)^* \ 1$
- Accepts string   0 1 1 0 1

Non-deterministic FSA's

- NFSA generalize DFSA I two ways:
- Include edges labeled by ε
  - Allows process to non-deterministically change state
- Each state can have zero, one or more edges labeled by each letter
  - Given a letter, non-deterministically choose an edge to use

NFSA Language Recognition

- Play the same game as with DFSA

- Free move: move across an edge with empty string label without discarding card

- When you run out of letters, if you are in final state, you win; string is in language

- You can take one or more moves back and try again

- If have tried all possible paths without success, then you lose; string not in language

FSA Language Recognition

- Move the disc from one state to next if edge between labeled the same as top card in deck; discard top card

- Free move: move across an edge with empty string label without discarding card

- When you run out of cards, if you are in final state, you win; string is in language

- You can take a move back and try another

- If you have cards left and you have tried all possible edges without success, then you lose; string not in language

Things to remember

- State diagrams model how state changes with actions
  - Useful for modeling sequences of events, complex probabilistic outcomes, planning, automation, etc.
  - Especially useful when states can be visited more than once